**Topic 3: Discrete and Continuous Probability Distributions Solutions**

**Q1**

a)  



b) Probability distribution of weight

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Weight | 100 | 110 | 120 | 130 | 140 |
| Probability | 0.1 | 0.3 | 0.4 | 0.1 | 0.1 |

c) Expected Weight ==118 lb

d) Standard deviation = = 10.77lb

e) Take Age= 21 and Weight= 110 lb as example,

Pr (Age=21 and Weight = 110) =0.06

Pr (Age=21) = 0.02+0.06+0.11+0.04+0.05 = 0.28

Pr (Weight=110) = 0.3

Pr (Weight=110) Pr (Age=21) 

Age and Weight are not independent.

**Q2**

a) Mean = 00.07+10.15+20.1+30.05 =0.5

Standard Deviation = 

= 0.8660

b) Expected total number == 1000.5 = 50

**Q3**

1. Distribution A:

 

Distribution B:



1. Distribution A:



Distribution B:



1. Distribution A and B has the same spread but locate at different position. Distribution A is on the left-hand-side of Distribution B.

**Q4**

1. Stock X:

expected return = (-50)(0.1)+(20)(0.3)+(100)(0.4)+(150)(0.2)= 71  
s.d.of return = = 61.88

Stock Y:

expected return = (-100)(0.1)+…+(200)(0.2) = 97

s.d. of return = = 84.27

b) Stock Y gives investor higher expected return than stock X., but also a higher standard deviation. Thus, a risk-averse investor should invest in stock X, while investor who is willing to take a higher risk can expect a higher return from stock Y.

**Q5**

a) *X* = no. of customers that the AIS detects as having exceeded their credit limit

*π* = success probability = 0.05

n = 20

∵ *X* is binomial distribution X ~B(n=20, *π* =0.05)

mean = n *π* = 20(0.05) = 1

variance = n *π* (*1 - π* ) = 20(0.05)(0.95) = 0.95

standard deviation =  = 0.9747

b) P(*X* = 0) =  = 0.3585

c) P(*X* = 1) =  = 0.3774

1. P(*X* ≥ 2) = 1 – P(X =0) – P(X = 1) = 1 – 0.3585 – 0.3774 = 0.2642

**Q6**

Let X be the number of customers who will leave the site without completing a transaction. X~B(20, 0.88)

P(X=20) = (0.88)(1-0.88)= 0.0776

**Q7**

a) P(did well or went shopping) = 0.95

b) Binomial distribution is used since

* + 1. no. of trials is fixed
    2. two mutually exclusive outcomes
    3. independent trials
    4. probability of success is constant

Let *Y* be the no. of students did well on mid-term test and studied for mid-term test the weekend before the mid-term test out of the selected 10 students

*Y ~ B(n = 10, π = 0.45)*



**Q8**

1. Pr(wine) = 1 – 0.7 = 0.3

b) Pr(beer and male) = 0.8 \*0.6 = 0.48

=> Pr(wine and male)= 0.6- 0.48 = 0.12

=> Pr(wine and female) = 0.3-0.12 = 0.18

c) Pr(male | wine) = Pr(male and wine) / Pr(wine) = 0.12/0.3 = 0.4

d) Define X be the number of patrons prefer beer in the 5 selected patrons,

X~B(5, 0.7) Binomial: n=5, *π*=0.7

Pr (at least 4 patrons) = Pr (x=4) +Pr(x=5)

= 5!/(4! 1!) 0.740.31 + 5!/(5! 0!) 0.750.30

= 0.36015 + 0.16807

= 0.52822

**Q9**

X is the number of passengers responded to the survey

π is the population proportion of passenger responded to the survey

π = 1 - 0.87 = 0.13

X ~ B(15, 0.13)

Pr(X ≥ 2) = 1 – Pr(X=0) – Pr(X=1)

= 1 – –

=1 – 0.1238 – 0.2275

=0.5987

**Q10**

a) P(x<2) = 0.0102+0.0768 = 0.087

b) E(X) = 0x0.0102+1x0.0768+2x0.2304+3x0.3456+4x0.2592+5x0.0778 = 3.0002

It means, on average, among 5 dentists 3 of them will use “laughing gas”.

c) V(X) = (0-3)2x0.0102+(1-3) )2x0.0768+(2-3) )2x0.2304++(3-3) )2x0.3456

+(4-3) )2x0.2592+(5-3) )2x0.0778

= 1.1998

Thus, SD(x) =

d) Define “success” = use laughing gas, “failure” = not use laughing gas, then X represents the number of “success” in n independent trials and the probability of success in each trial is p. According to results of b and c, we have nπ = 3 and nπ (1-π) = 1.1998. Solve the equations, we have n = 5 and π = 0.6.